



Lecture8 Single-mode squeezing

①

high energy (pump) ω_1 low energy ω_2 low energy

parametric excitation

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \rightarrow \nabla^2 \vec{E} - \mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \quad v = \frac{1}{\sqrt{\mu \epsilon}}$$

②

$$\vec{P} = \epsilon_0 \chi^{(1)} \vec{E}$$

$$\nabla^2 \vec{E} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} = 0$$

General

$$\vec{P} = \epsilon_0 (\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E} \vec{E} + \chi^{(3)} \vec{E} \vec{E} \vec{E})$$

3x3 tensor 2x2x2

$\chi^{(1)}$ $\chi^{(2)}$ $\chi^{(3)}$

$\vec{P} = \epsilon_0 (\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E} \vec{E} + \chi^{(3)} \vec{E} \vec{E} \vec{E})$

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$\omega = ck_0 = vk$

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \chi^{(1)} \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \epsilon_0 \chi^{(2)} \frac{\partial^2 \vec{E}^2}{\partial t^2} = 0$$

Wave Source

parametric process (wave-mixing)

$$\vec{E}(\vec{r}, t) = e^{i\vec{k}\cdot\vec{r}} E(t)$$

$$-k^2 \vec{E} - \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \epsilon_0 \chi^{(2)} \frac{\partial^2 \vec{E}^2}{\partial t^2}$$

$$-\omega^2 \vec{E} - \frac{1}{\epsilon} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\epsilon} \chi^{(2)} \frac{\partial^2 \vec{E}^2}{\partial t^2}$$


harmonic oscillator

drive

3

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E - \frac{1}{\hbar^2} \chi^{(2)} \frac{\partial^2 E^2}{\partial t^2}$$

$$\frac{\partial^2 E_1}{\partial t^2} = -\omega^2 E_1 - \frac{1}{\hbar^2} \chi^{(2)} \frac{\partial^2 (\dots)}{\partial t^2}$$

$$\frac{\partial^2 E_2}{\partial t^2} = -\omega^2 E_2 - \frac{1}{\hbar^2} \chi^{(2)} \frac{\partial^2 (\dots)}{\partial t^2}$$

$$E = E_1 + E_2 + E_p$$

$$E = \underbrace{E_1 + E_2}_{\text{two-mode}} + E_p$$

$$E^2 = (E_1 + E_2 + E_p)(E_1 + E_2 + E_p)$$

$$= E_1^2 + E_2^2 + E_p^2 + 2E_1E_2 + 2E_1E_p + 2E_2E_p$$

$$\frac{\partial^2 E_1}{\partial t^2} = -\omega_1^2 E_1 - \frac{1}{n^2} \chi^{(2)} \frac{\partial^2 (\dots)}{\partial t^2}$$

$$E_1 = E^{(0)} + \xi E^{(1)} + \xi^2 E^{(2)} + \dots \quad (\xi \ll 1)$$

$$\frac{\partial^2 E_1^{(0)}}{\partial t^2} = -\omega_1^2 E_1^{(0)}$$

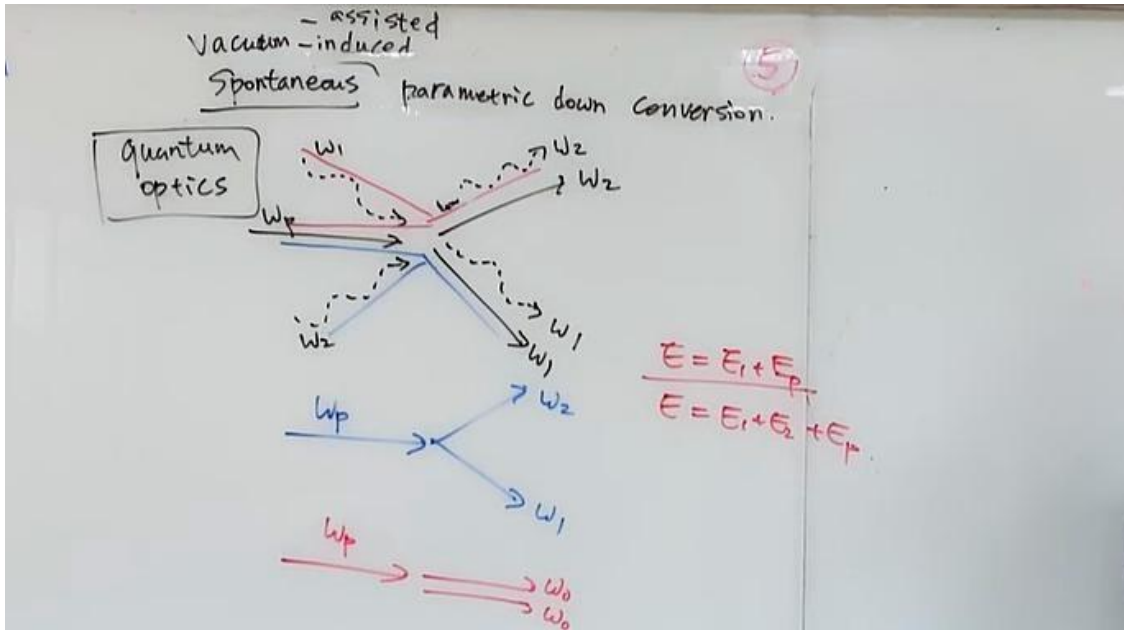
$$\xi \frac{\partial^2 E_1^{(1)}}{\partial t^2} = -\omega_1^2 \xi E_1^{(1)} - \frac{1}{n^2} \chi^{(2)} \frac{\partial^2 (E_1^{(0)2} + E_2^{(0)2} + E_1^{(0)} E_2^{(0)} + E_1^{(0)2} + E_2^{(0)2} + E_p^{(0)2})}{\partial t^2}$$

$$\frac{\partial^2 E_1^{(1)}}{\partial t^2} = -\omega_1^2 E_1^{(1)} - \frac{1}{n^2} \frac{\chi^{(2)}}{\xi} \frac{\partial^2 (E_1^{(0)} E_p^{(0)})}{\partial t^2}$$

Diagrams illustrating wave interactions and energy levels:

- Left: Energy level diagram showing transitions between $E_1^{(0)}$ and $E_2^{(0)}$ states, with labels ω_1 , ω_p , and ω_2 .
- Right: Wave vector diagram showing the relationship between ω_p , ω_1 , and ω_2 , with a box indicating $\omega_p \approx \omega_2$ and ω_1 .





$E^2 + B^2 = \frac{1}{\epsilon_0} \hat{E} \cdot \hat{E} + \frac{1}{\epsilon_0} \hat{B} \cdot \hat{B}$
 $\hat{E} = \frac{1}{\epsilon_0} \hat{a} + \hat{a}^\dagger$
 $\hat{B} = \frac{1}{\epsilon_0} \hat{a} - \hat{a}^\dagger$
 $\hat{H} = \hbar \omega_0 \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \left(\hat{a} \cdot \hat{E} \right)$
 $\hat{H} = \hbar \omega_0 \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \epsilon_0 \chi^{(1)} \hat{E} + \epsilon_0 \chi^{(2)} \hat{E}^2 + \epsilon_0 \chi^{(3)} \hat{E}^3$
 $\hat{H} = \hbar \omega_0 \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \left(\eta \hat{a}^2 + \eta^* \hat{a}^{\dagger 2} \right) e^{i\omega_0 t} + \left(\alpha \hat{a} + \alpha^* \hat{a}^\dagger \right) e^{i\omega_0 t}$
 $\hat{H}_{\text{eff}} = \left(\eta \hat{a}^2 + \eta^* \hat{a}^{\dagger 2} \right) e^{i\omega_0 t} + \left(\alpha \hat{a} + \alpha^* \hat{a}^\dagger \right) e^{i\omega_0 t}$
 $\hat{H}_{\text{eff}} = \eta \hat{a}^2 + \eta^* \hat{a}^{\dagger 2}$
 $\frac{i\hbar \partial |\psi\rangle}{\partial t} = \hat{H}_{\text{eff}} |\psi\rangle \rightarrow |\psi\rangle = e^{-\frac{i\hat{H}_{\text{eff}}}{\hbar} t} |\psi(0)\rangle = e^{-\frac{i}{\hbar} (\eta \hat{a}^2 + \eta^* \hat{a}^{\dagger 2}) t} |\psi(0)\rangle$
 $\hat{D}(\alpha) |\psi\rangle = \alpha |\psi\rangle$

