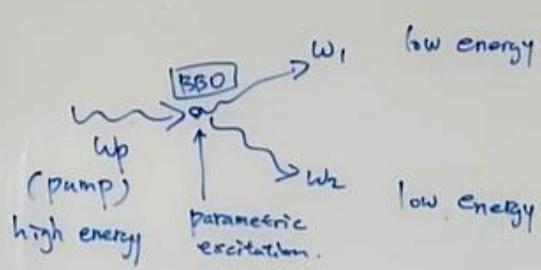


Lecture8 Single-mode squeezing

①



$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \rightarrow \nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \quad v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

②

$$\vec{P} = \epsilon_0 \chi^{(1)} \vec{E}$$

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{P}}{\partial t^2} = 0$$

$$\vec{P} = \epsilon_0 (\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E}^2 + \chi^{(3)} \vec{E}^3)$$

$$\vec{P} = \epsilon_0 \left(\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E} \vec{E} + \chi^{(3)} \vec{E} \vec{E} \vec{E} \right)$$

$$P_1, P_2, P_3$$

$$\chi_{ijk} \quad \text{tensor } 3 \times 3 \times 3$$

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \epsilon_0 \chi^{(2)} \frac{\partial^2 \vec{E}^2}{\partial t^2}$$

$$-k^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \epsilon_0 \chi^{(2)} \frac{\partial^2 \vec{E}^2}{\partial t^2}$$

$$-\omega^2 \vec{E} - \frac{1}{\omega^2} \vec{E} = \frac{1}{\omega^2} \chi^{(2)} \frac{\partial^2 \vec{E}^2}{\partial t^2}$$

$$\vec{E}(k, \vec{r}) e^{i k \vec{r}} E(t)$$

$$\omega = c k_0 = v k$$



parametric excitation (3)

harmonic oscillator

$$\frac{\partial^2 E}{\partial t^2} = -\omega E - \frac{1}{h^2} \chi^{(2)} \frac{\partial^2 E^2}{\partial t^2}$$

drive

$$\frac{\partial^2 E_1}{\partial t^2} = -\omega_1 E_1 - \frac{1}{h^2} \chi^{(2)} \frac{\partial^2 (\dots)}{\partial t^2}$$

$$\frac{\partial^2 E_2}{\partial t^2} = -\omega_2 E_2 - \frac{1}{h^2} \chi^{(2)} \frac{\partial^2 (\dots)}{\partial t^2}$$

$$E = E_1 + E_2 + E_p$$

single-mode

two-mode

$$E^2 = (E_1 + E_2 + E_p)(E_1 + E_2 + E_p)$$

$$= E_1^2 + E_2^2 + E_p^2 + 2E_1 E_2 + 2E_1 E_p + 2E_2 E_p$$

(4)

$\frac{\partial^2 E_1}{\partial t^2} = -\omega_1^2 E_1 - \frac{1}{h^2} \chi^{(2)} \frac{\partial^2 (\dots)}{\partial t^2}$

$E_1 = E^{(0)} + \xi E^{(1)} + \xi^2 E^{(2)} + \dots$ (non-resonant)

$\frac{\partial^2 E_1^{(0)}}{\partial t^2} = -\omega_1^2 E_1^{(0)}$

$\frac{\partial^2 E_1^{(1)}}{\partial t^2} = -\omega_1^2 \xi E_1^{(1)} - \frac{1}{h^2} \chi^{(2)} \frac{\partial^2 (E_1^{(0)} E_p + E_2^{(0)} E_p + E_1^{(0)} E_2 + E_1^{(0)} + E_2^{(0)} + E_p^2)}{\partial t^2}$

$\frac{\partial^2 E_1^{(2)}}{\partial t^2} = -\omega_1^2 E_1^{(2)} - \frac{1}{h^2} \chi^{(2)} \frac{\partial^2 (E_1^{(0)} E_2^{(0)})}{\partial t^2}$

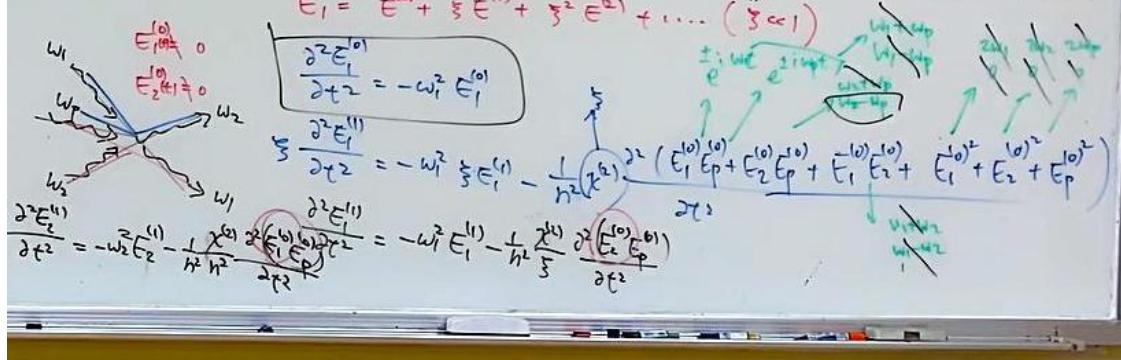
$\omega_p - \omega_2$

ω_1

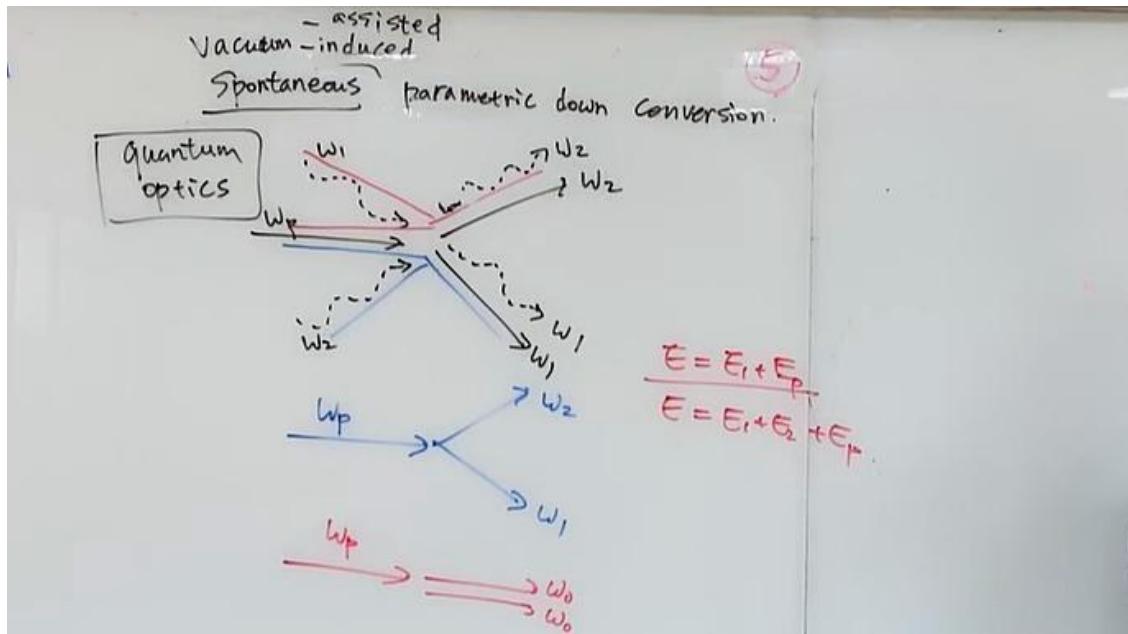
ω_2

$\omega_p - \omega_1$

Non-resonant







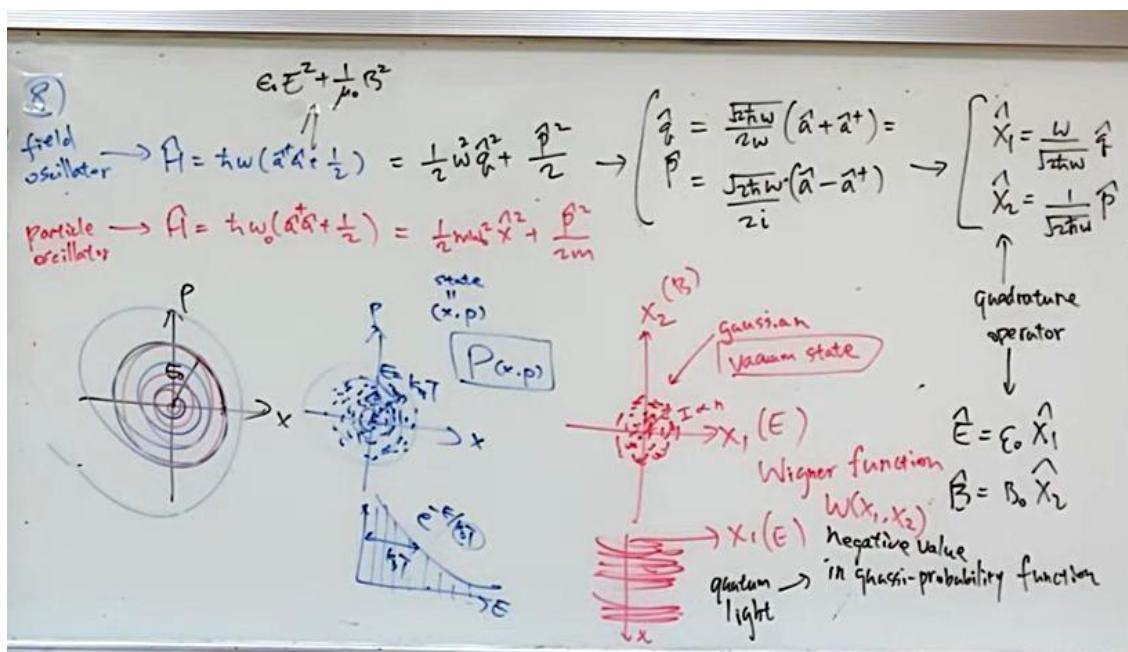
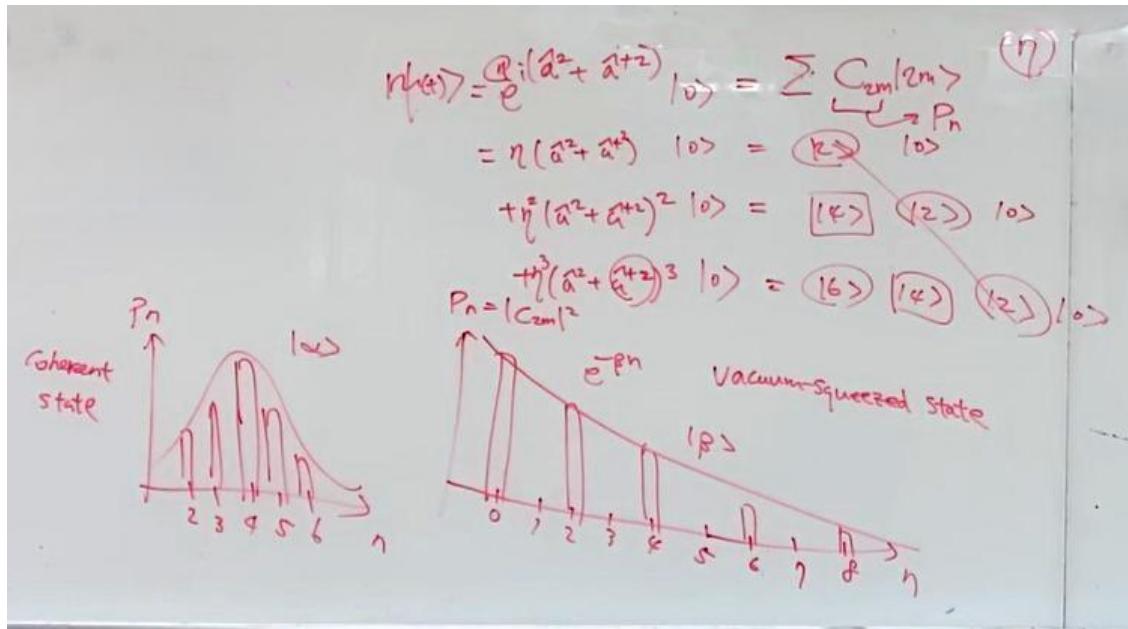
b)

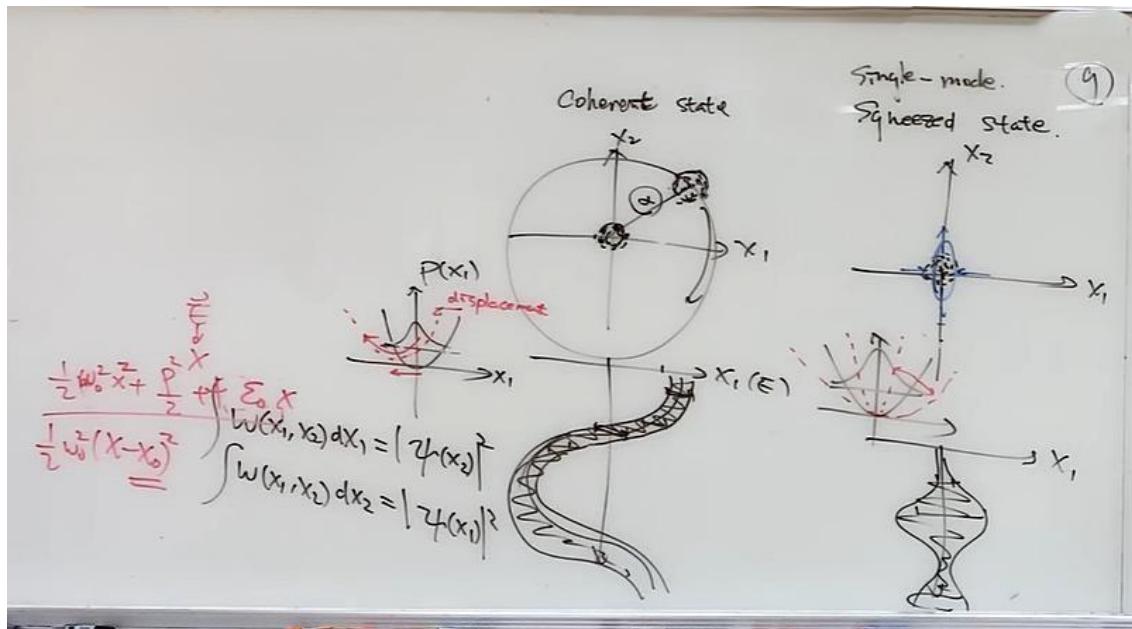
$$\begin{aligned} E^2 + B^2 & \stackrel{1}{\downarrow} \quad \stackrel{1}{\stackrel{E = \frac{a+a^*}{2}}{\stackrel{\downarrow}{B = \frac{a-a^*}{2i}}}} \\ \hat{H} &= \hbar \omega_0 (a^* a + \frac{1}{2}) + \boxed{\vec{d} \cdot \vec{E}_0} \\ \hat{H} &= \hbar \omega_0 (a^* a + \frac{1}{2}) \\ d &= \epsilon_0 \chi^{(2)} E + \epsilon_0 \chi^{(2)} E^2 + \epsilon_0 \chi^{(2)} E^3 \\ &+ (a^2 + \eta^* a^{*2}) e^{i4\phi} + a^* e^{-i4\phi}) E = E_1 + E_p \\ \rightarrow \hat{H}_{\text{int}} &= (\eta a^2 e^{i2\phi} + \eta^* a^{*2} e^{-i2\phi}) \epsilon_0 \chi^{(2)} E_1 E_p \\ \rightarrow \hat{H}_{\text{int}} &= \eta a^2 + \eta^* a^{*2} \\ \frac{\partial \hat{H}_{\text{int}}}{\partial t} = \hat{H}_{\text{int}} & \rightarrow |4\rangle = e^{-\frac{i}{\hbar} \hat{H}_{\text{int}} t} |4(0)\rangle = e^{-\frac{i}{\hbar} (\eta a^2 + \eta^* a^{*2}) t} |4(0)\rangle \end{aligned}$$

parametric approximation

$$\begin{aligned} \hat{a}_p |a e^{i4\phi}\rangle &= \underline{a e^{i4\phi}} |a e^{i4\phi}\rangle \\ \hat{a}^* |a e^{i4\phi}\rangle &\approx \underline{a e^{-i4\phi}} |a e^{i4\phi}\rangle \\ \hat{D}(\omega) &= \frac{1}{\hbar} (\hat{a} + \hat{a}^*) \\ \hat{D}(a) |b\rangle &= a |b\rangle \end{aligned}$$

displacement operator $\hat{D}(\omega)$

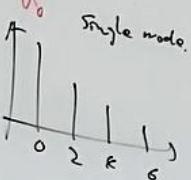




(10) two-mode squeezing

$$A = \hbar w_a (\hat{a}_a^\dagger \hat{a}_a + \frac{1}{2}) + \hbar w_b (\hat{a}_b^\dagger \hat{a}_b + \frac{1}{2}) + (\eta (\hat{a}_a^\dagger \hat{a}_b^\dagger + \eta^\dagger \hat{a}_a^\dagger \hat{a}_b))$$

Single mode:



$\hat{a}_a |0_a, 0_b\rangle = e^{i(\eta \hat{a}_a \hat{a}_b + \eta^\dagger \hat{a}_a^\dagger \hat{a}_b^\dagger)} |0_a, 0_b\rangle$

$$\hat{a}_a |0_a, 0_b\rangle = \sum_{n_1, n_2} c_{n_1 n_2} |n_1, n_2\rangle$$

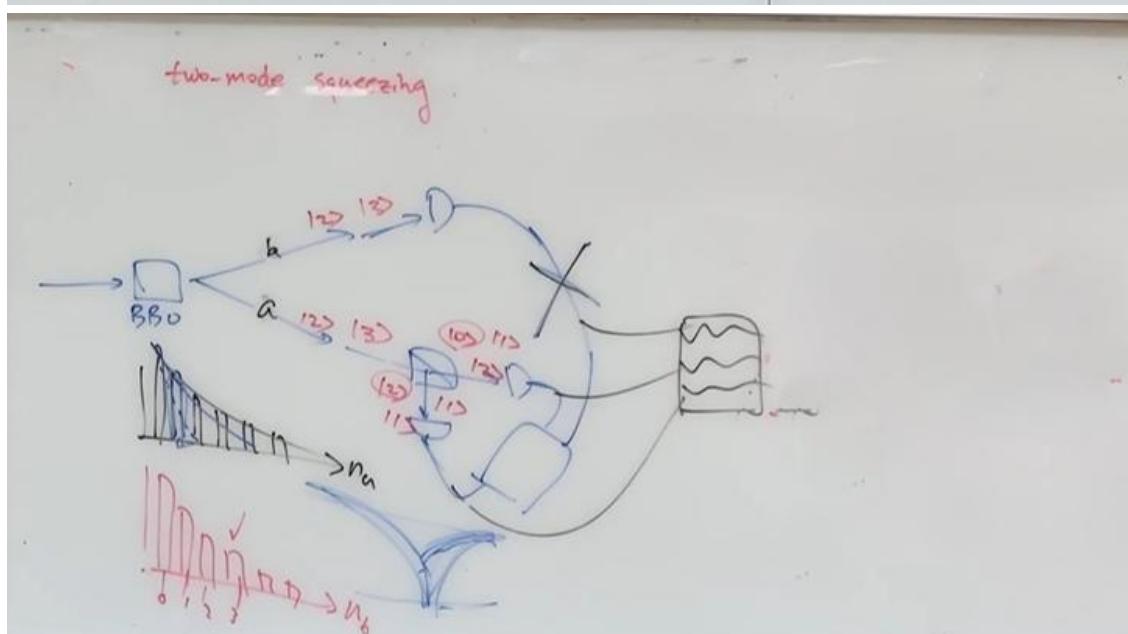
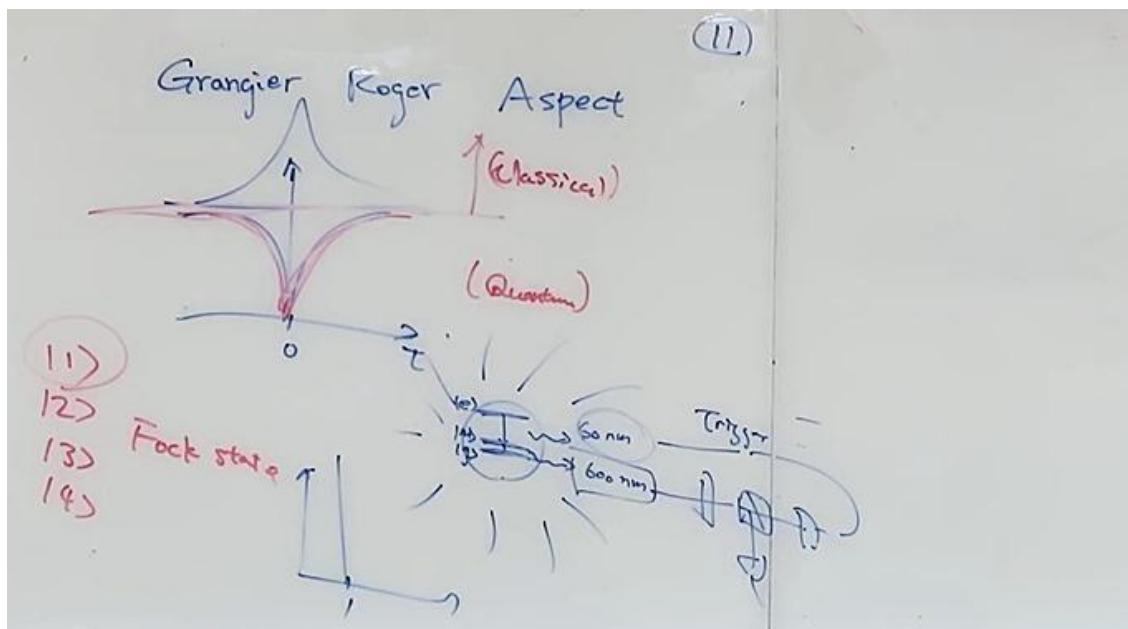
$$\hat{a}_a^\dagger |0_a, 0_b\rangle = \sum_{n_1, n_2} c_{n_1 n_2}^\dagger |n_1, n_2\rangle$$

$$|0_a, 0_b\rangle = \hat{a}_a |0_a, 0_b\rangle + \hat{a}_a^\dagger |0_a, 0_b\rangle$$

$$|0_a, 0_b\rangle = (\hat{a}_a \hat{a}_b + \hat{a}_a^\dagger \hat{a}_b^\dagger)^2 |0_a, 0_b\rangle$$

$$|0_a, 0_b\rangle = (\hat{a}_a \hat{a}_b + \hat{a}_a^\dagger \hat{a}_b^\dagger)^3 |0_a, 0_b\rangle$$







- ✓ - parametric excitation
- ✓ - Single-mode squeezing
- ✓ - two-mode squeezing
- ✓ - GRA experiment

